[8] R. F. Millar, "On the Rayleigh assumption in scattering by a periodic surface," *Proc. Cambridge Phil. Soc.*, vol. 65, pp. 773–791, 1969.
[9] ——, "Singularities of two-dimensional exterior solutions of the Helmholtz equations," *Proc. Cambridge Phil. Soc.*, vol. 69, pp. 175–188, 1971.
[10] ——, "On the Rayleigh assumption in scattering by a periodic surface. II," *Proc. Cambridge Phil. Soc.*, pp. 217–225, 1971.
[11] ——, "The location of singularities of two-dimensional harmonic functions. I: Theory," *SIAM J. Math. Anal.*, vol. 1, pp. 333–344, Aug. 1970.
[12] ——, "The location of singularities of two-dimensional harmonic functions. II: Applications," *SIAM J. Math. Anal.*, pp. 345–353, Aug. 1970.
[13] G. N. Watson, *A Treatise on the Theory of Bessel Functions*, 2nd ed. New York: Cambridge, 1958, ch. 3.
[14] J. D. Hunter and R. H. T. Bates, "Secondary diffraction from close edges on perfectly conducting bodies," *Int. J. Electron.*, in press.
[15] J. D. Hunter, "The surface current density on perfectly conducting polygonal cylinders," *Can. J. Phys.*, in press.
[16] P. Silvester, "A general high-order finite-element waveguide analysis program," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 204–210, Apr. 1969.
[17] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1965.
[18] P. J. Davies and P. Rabinowitz, *Numerical Integration*. Waltham, Mass.: Blaisdell, 1967.
[19] L. Lewin, "On the restricted validity of point-matching techniques," *IEEE Trans. Microwave Theory Tech.* (*1970 Symposium Issue*), vol. MTT-18, pp. 1041–1047, Dec. 1970.
[20] R. M. Bulley and J. B. Davies, "Computation of approximate polynomial solutions to TE modes in an arbitrarily shaped waveguide," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Computer-Oriented Microwave Practices*), vol. MTT-17, pp. 440–446, Aug. 1969.

Broadside-Coupled Strips in a Layered Dielectric Medium

JAMES L. ALLEN AND MARVIN F. ESTES

Abstract—**Coupling coefficient, odd- and even-mode impedances, and the mode phase velocities are calculated for a pair of broadside-coupled strips embedded in a layered dielectric medium and enclosed by a rectangular shield. Large ratios of mode phase velocities can be achieved with this structure leading to novel performance characteristics for familiar coupled-line configurations such as the microwave *C* section.**

Introduction

COUPLED-LINE structures are utilized extensively as building blocks for directional couplers, filters, and other important transmission-line devices. Coupled lines in a homogeneous medium have equal even- and odd-mode phase velocities, but the corresponding velocities are unequal for coupled lines in an inhomogeneous medium. Structures of both types have been treated in the literature [1]–[6]. The configurations that have been studied most extensively are stripline and microstrip structures with nearly equal even- and odd-mode phase velocities (ratios typically less than 1.3). Deviation from equal velocities has generally been regarded as undesirable. However, as demonstrated by Dalley [7], configurations with large velocity ratios can be used to advantage. The purpose of this paper is to present design data for a coupled-line structure capable of realizing large phase-velocity ratios.

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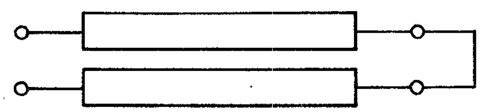

Fig. 1. Schematic representation of a *C* section.

As an illustration of the interesting changes that can occur in the performance of familiar devices when odd- and even-mode phase velocities are significantly different, consider the microwave *C* section shown schematically in Fig. 1. For a line with equal even- and odd-mode velocities the *C* section is an all-pass structure [8]. The response of an inhomogeneous *C* section is distinctly bandpass. The *ABCD* or chain matrix representation for the inhomogeneous *C* section can be obtained by the method of Jones and Bolljahn [9]. The result is

$$\begin{bmatrix} V_{in} \\ I_{in} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{out} \\ -I_{out} \end{bmatrix} \tag{1}$$

where

$$A = D = \frac{1}{\Delta}(Z_{0e}\cot\theta_e - Z_{0o}\tan\theta_o),$$

$$B = \frac{2j}{\Delta} Z_{0e}Z_{0o}\cot\theta_e\tan\theta_o,$$

$$C = \frac{2j}{\Delta},$$

$$\Delta = Z_{0e}\cot\theta_e + Z_{0o}\tan\theta_o,$$

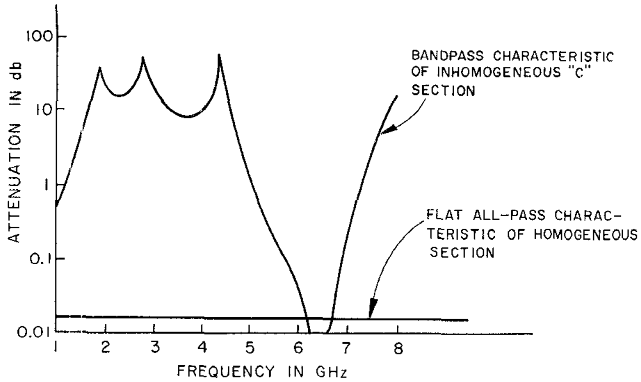

Fig. 2. Response of an inhomogeneous C section. ($v_e/v_o=1.9$, $Z_{0o}=25$ Ω, $Z_{0e}=110$ Ω, coupled section length $=0.625$ cm).

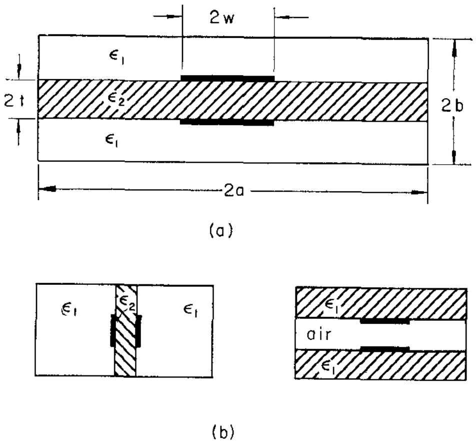

Fig. 3. (a) Broadside-coupled strips in a layered dielectric medium. Center-strip thickness is zero. (b) Variations on the structure shown in part (a).

$\theta_e=\beta_e l$ even-mode electrical length of line,
$\theta_o=\beta_o l$ odd-mode electrical length of line,
Z_{0o} odd-mode impedance,
Z_{0e} even-mode impedance.

In terms of the $ABCD$ parameters the insertion ratio for a network terminated by Z_L and driven by a generator with impedance Z_G is

$$\text{insertion ratio} = \frac{AZ_L + CZ_LZ_G + B + DZ_G}{Z_L + Z_G}. \quad (2)$$

The response of a C section with a phase-velocity ratio of 1.9, odd- and even-mode impedances 25 and 110 Ω, respectively, a coupled section length of 0.625 cm, and $Z_L=Z_G=50$ Ω is shown in Fig. 2. The bandpass response is quite pronounced with three peaks in attenuation for the single section.

The basic configuration, shown in Fig. 3, consists of a pair of broadside-coupled strips embedded in a layered dielectric medium and enclosed in a rectangular shield. Coupling coefficient, odd- and even-mode impedances, and the mode phase velocities are calculated for a variety of material and structural parameters. The results are presented in the form of design curves. The homogeneous case ($\epsilon_2=\epsilon_1$) has been studied earlier by Krage and Haddad [4], and the case where $\epsilon_1=1$, $\epsilon_2=2.35$ with the strips excited in phase (even mode) corresponds to the dielectric-supported air stripline with side walls studied recently by Gish and Graham [10].

Formulation of the Problem

The layered dielectric precludes pure TEM-mode propagation. It is assumed, however, that the longitudinal components of the E- and H-fields are so small compared to the transverse field that they can be neglected. This approximation, which has been shown [6] to give excellent results for coupled microstrip up to frequencies of several gigahertz, is used throughout the following analysis. On the basis of the quasi-TEM assumption the calculation of impedances, phase velocities, and coupling coefficient reduces, essentially, to a solution of the two-dimensional Laplace's equation $\nabla^2\Phi(x, y)=0$, subject to boundary conditions determined by the geometry of the line. The method of Gish and Graham [10] is employed here to calculate the odd- and even-mode capacitances. Impedances, phase velocities, and coupling coefficients are calculated from the capacitance values. The odd- and even-mode impedances and velocities and the coupling coefficient can be expressed in terms of the capacitances as

$$Z_{0o} = \frac{1}{v_f\sqrt{C_{0o}{}^f C_{0o}{}^d}} = \text{odd-mode impedance} \quad (3)$$

$$Z_{0e} = \frac{1}{v_f\sqrt{C_{0e}{}^f C_{0e}{}^d}} = \text{even-mode impedance} \quad (4)$$

$$v_o = v_f\sqrt{\frac{C_{0o}{}^f}{C_{0o}{}^d}} = \text{odd-mode velocity} \quad (5)$$

$$v_e = v_f\sqrt{\frac{C_{0e}{}^f}{C_{0e}{}^d}} = \text{even-mode velocity} \quad (6)$$

$$k = \frac{\sqrt{C_{0o}{}^f C_{0o}{}^d} - \sqrt{C_{0e}{}^f C_{0e}{}^d}}{\sqrt{C_{0o}{}^f C_{0o}{}^d} - \sqrt{C_{0e}{}^f C_{0e}{}^d}} = \text{coupling coefficient} \quad (7)$$

where v_f = velocity of light in free space,

$$\begin{pmatrix} C_{0e}{}^d \\ C_{0o}{}^d \end{pmatrix} = \begin{pmatrix} \text{even} \\ \text{odd} \end{pmatrix}$$ mode capacitance per unit length with the layered dielectric in place

$$\begin{pmatrix} C_{0e}{}^f \\ C_{0o}{}^f \end{pmatrix} = \begin{pmatrix} \text{even} \\ \text{odd} \end{pmatrix}$$ mode capacitance per unit length with the dielectric layers replaced by free space (i.e., $\epsilon_2 = \epsilon_1 = 1$).

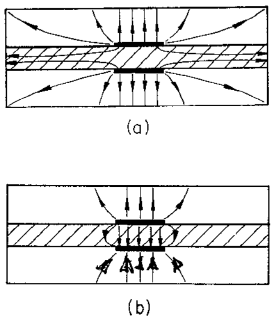

Fig. 4. (a) Sketch of electric-field lines for even mode. (b) Sketch of electric-field lines for odd mode.

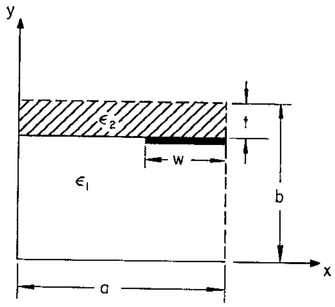

Fig. 5. Quarter section analyzed. Center-strip thickness is zero.

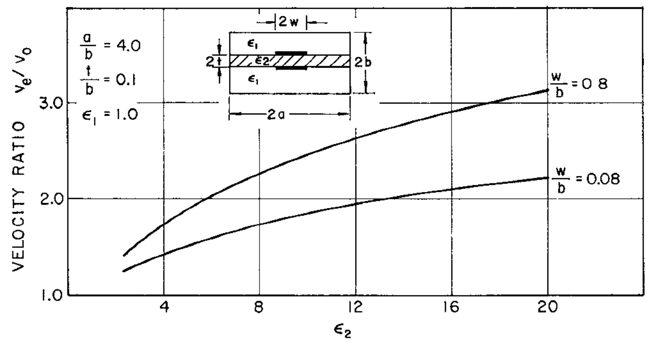

Fig. 6. Ratio of even- to odd-mode phase velocity as a function of ϵ_2 [$(a/b)=4$, $(t/b)=0.1$, $\epsilon_1=1.0$].

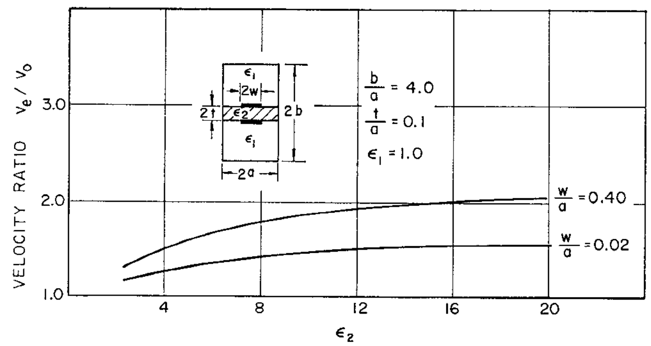

Fig. 7. Ratio of even- to odd-mode phase velocity as a function of ϵ_2 [$(a/b)=\frac{1}{4}$, $(t/a)=0.1$, $\epsilon_1=1.0$].

Capacitance Solutions

Sketches of the electric-field lines for the odd and even modes are shown in Fig. 4. In view of the symmetry of the configuration, the field solution is required for only one quarter of the cross section, as shown in Fig. 5. The total capacitance for each mode is simply four times that obtained for the quarter section.

Each capacitance is calculated using the following lower bound variational expression as given in [11, p. 194]:

$$C_o = \frac{\left[\int_{a-w}^{a} \sigma(x)\, dx\right]^2}{\int_{a-w}^{a}\int_{a-w}^{a} G_o(x, \xi, d, d)\sigma(x)\sigma(\xi)d\xi\, dx} \tag{8}$$

where

G_o Green's function,

σ charge density on center strip.

The integral limits and variable names have been adapted to the present problem. For the even mode the Green's function is given by [10, eq. (18)]. The Green's function for the odd mode can also be determined by the method used by Gish and Graham and is found to be

$$G_{0o}(x, \xi, y, d) = \frac{2}{\pi\epsilon_0}\begin{cases} \displaystyle\sum_{n=1,3,5,\cdots}^{\infty}\left[\frac{\sin\gamma_n y \sinh\gamma_n(b-d)\sin\gamma_n x\sin\gamma_n\xi}{\frac{n}{2}\left[\epsilon_1\cosh\gamma_n d\sinh\gamma_n(b-d)+\epsilon_2\cosh\gamma_n(b-d)\sinh\gamma_n d\right]}\right], & \text{for } 0\le y\le d\\ \displaystyle\sum_{n=1,3,5,\cdots}^{\infty}\left[\frac{\sin\gamma_n d \sinh\gamma_n(b-y)\sin\gamma_n x\sin\gamma_n\xi}{\frac{n}{2}\left[\epsilon_1\cosh\gamma_n d\sinh\gamma_n(b-d)+\epsilon_2\cosh\gamma_n(b-d)\sinh\gamma_n d\right]}\right], & \text{for } d\le y\le b \end{cases} \tag{9}$$

where $\gamma_n=(n\pi/2a)$. The unknown charge density on the center strip for each mode is expanded in a cosine series. The approximate charge density for the even mode is given by [10, eq. (12)]. For the odd mode the corresponding expression is

$$\sigma_o(x) = \sum_{r=0}^{N} k_r \cos\left[\frac{2\pi r(x-a+w)}{2w}\right]. \tag{10}$$

Substituting the appropriate Green's function and charge density into (8) and performing the indicated integrations leads to expressions for the odd-

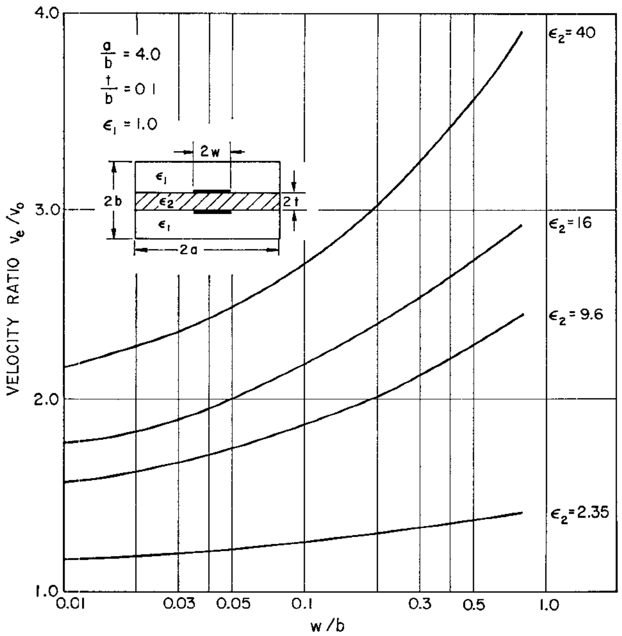

Fig. 8. Ratio of even- to odd-mode phase velocity as a function of (w/b) $[(a/b)=4,\ (t/b)=0.1,\ \epsilon_1=1.0]$.

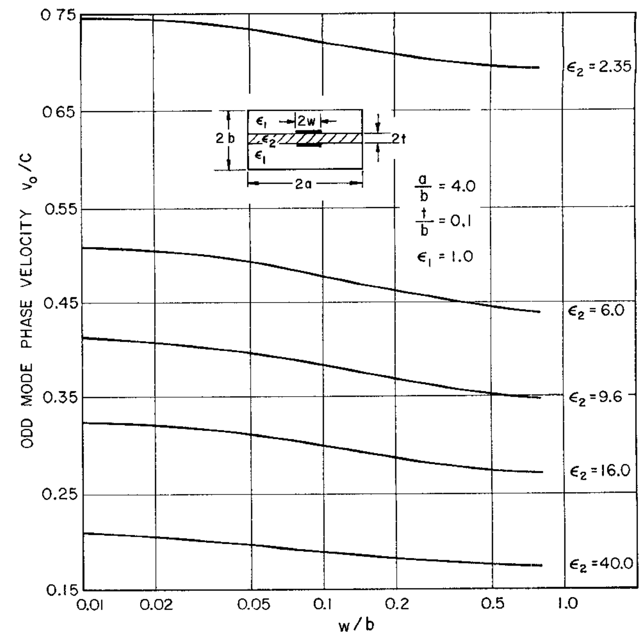

Fig. 9. Normalized odd-mode phase velocity as a function of (w/b) $[(a/b)=4,\ (t/b)=0.1,\ \epsilon_1=1.0]$.

and even-mode capacitances. The even-mode capacitance is given by [10, eq. (19)]. The odd-mode capacitance is

$$C_{0o}{}^{d} = \frac{\pi\epsilon_0}{2}\left\{\frac{1}{\sum_{r=0}^{N}\sum_{r=0}^{N} k_r k_s Q_{rs}}\right\} \tag{11}$$

where

$$Q_{rs} = \sum_{n=1,3,5,\cdots}^{\infty} \frac{\left(\frac{2an}{\pi w}\right)^2 B_n \sin^2 \gamma_n w}{\left[n^2 - \left(\frac{2ar}{w}\right)^2\right]\left[n^2 - \left(\frac{2as}{w}\right)^2\right]} \tag{12a}$$

and

$$B_n = \frac{\sinh \gamma_n d \sinh \gamma_n (b-d)}{\frac{n}{2}\left[\epsilon_1 \cosh \gamma_n d \sinh \gamma_n (b-d) + \epsilon_2 \cosh \gamma_n (b-d) \sinh \gamma_n d\right]}. \tag{12b}$$

The series for Q_{rs} and the corresponding series for the even mode are, in general, slowly converging. Alternate forms which converge rapidly were used to carry out the numerical calculations. These rapidly converging forms were obtained using the contour integration method described in [11, pp. 582–589].

At this point all parameters in equations for the capacitances can be evaluated except the k's and h's which are coefficients of the charge-density series. To solve for the k's, form the first partial derivatives $\partial C_{0o}{}^{d}/\partial k_r$) and set them equal to zero yielding the following set of N inhomogeneous equations:

$$\sum_{s=1}^{N} k_s Q_{rs} = -Q_{0s} \quad \text{for} \quad r = 1, 2, 3, \cdots, N. \tag{13}$$

This set of equations was solved using the Cholesky method as described in [12, ch. 5] programmed for a CDC 6400 digital computer. The values for the coefficients k_r from (13) were substituted into (11) and $C_{0o}{}^{d}$ evaluated. Various values of N were used. A value $N=22$ gave capacitance values accurate to one in the fourth place. The parameters for the even mode are determined in a similar manner. The values of $C_{0o}{}^{f}$ and $C_{0e}{}^{f}$ are obtained by setting $\epsilon_1=\epsilon_2=1.0$.

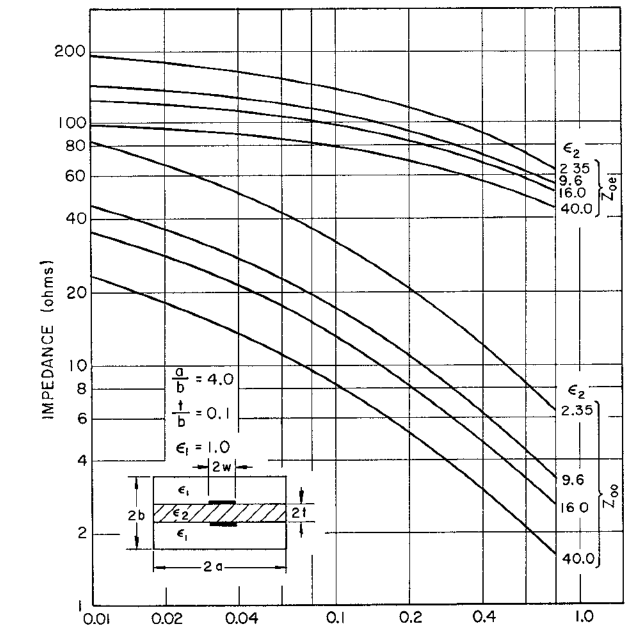

Fig. 10. Odd- and even-mode impedances as a function of (w/b) $[(a/b)=4,\ (t/b)=0.1,\ \epsilon_1=1.0]$.

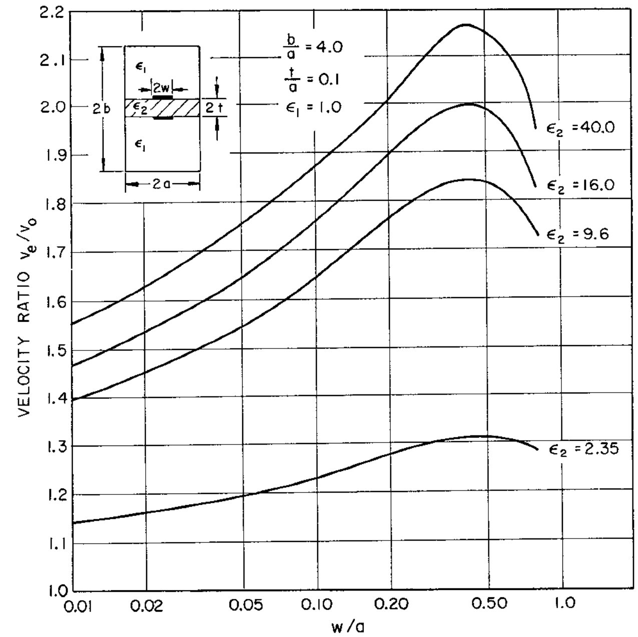

Fig. 12. Ratio of even- to odd-mode phase velocity as a function of (w/a) $[(a/b)=\frac{1}{4},\ (t/a)=0.1,\ \epsilon_1=1.0]$.

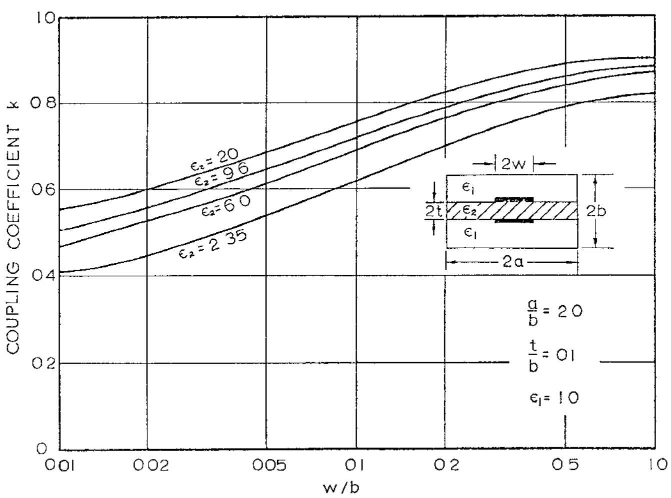

Fig. 11. Coupling coefficient as a function of (w/b) $[(a/b)=2,\ (t/b)=0.1,\ \epsilon_1=1.0]$.

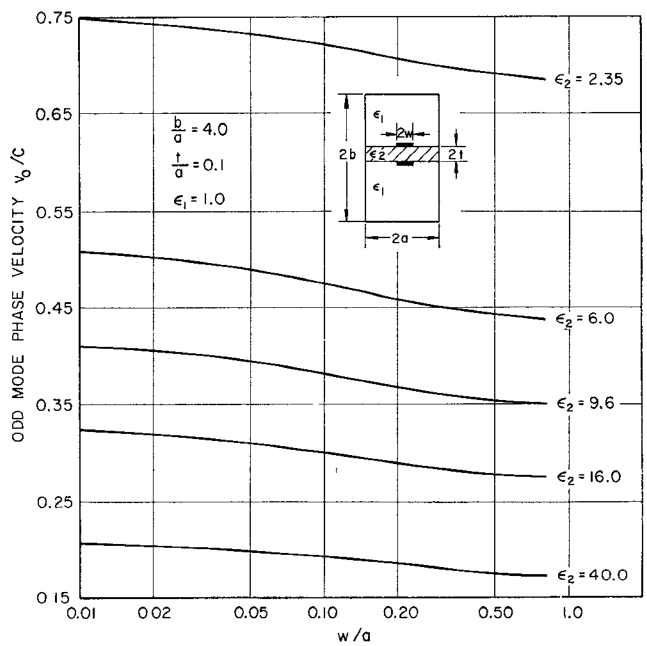

Fig. 13. Normalized odd-mode phase velocity as a function of (w/a) $[(a/b)=\frac{1}{4},\ (t/a)=0.1,\ \epsilon_1=1.0]$.

Numerical Results

By adjusting the parameters of the structure shown in Fig. 3, it is possible to achieve a wide range of mode phase velocities. If the dielectric constant of the center slab is greater than that of the upper and lower slabs, the even-mode velocity is greater than the odd-mode velocity. If the dielectric constant of the center slab is less than that of the other two slabs, the even-mode velocity is the smaller of the two. If all three dielectrics are the same, the odd- and even-mode velocities are equal. As shown in Figs. 6, 7, and 15, the velocity ratio v_e/v_o may be varied over the range $0.43 \leq (v_e/v_o) \leq 2.9$ using materials with dielectric constants less than 16. Due to fringe field effects, increasing the dielectric constant above 16 is less effective in increasing the velocity ratio, particularly for relatively narrow center strips.

Figs. 8–10 present phase-velocity ratio, odd-mode phase velocity, and odd- and even-mode impedances with the aspect ratio $(a/b)=4$, center-slab thickness $t=0.1\ b$, and air as the upper and lower dielectric layers. Increasing either the dielectric constant of the center

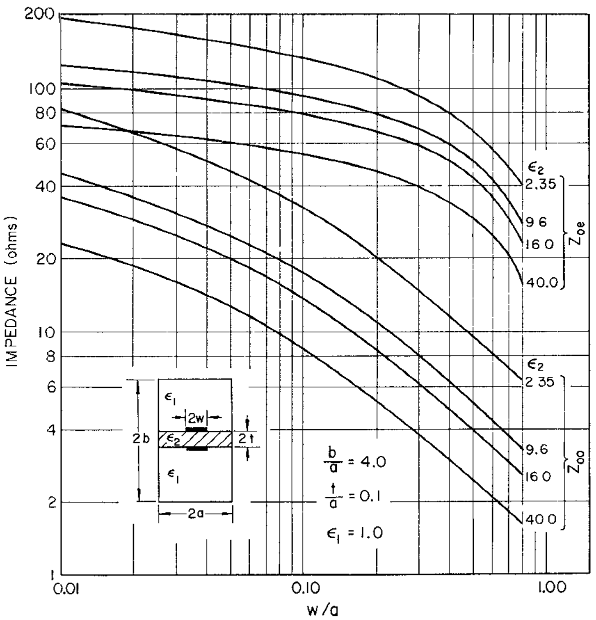

Fig. 14. Odd- and even-mode impedances as a function of (w/a) $[(a/b)=\frac{1}{4},\ (t/a)=0.1,\ \epsilon_1=1.0]$.

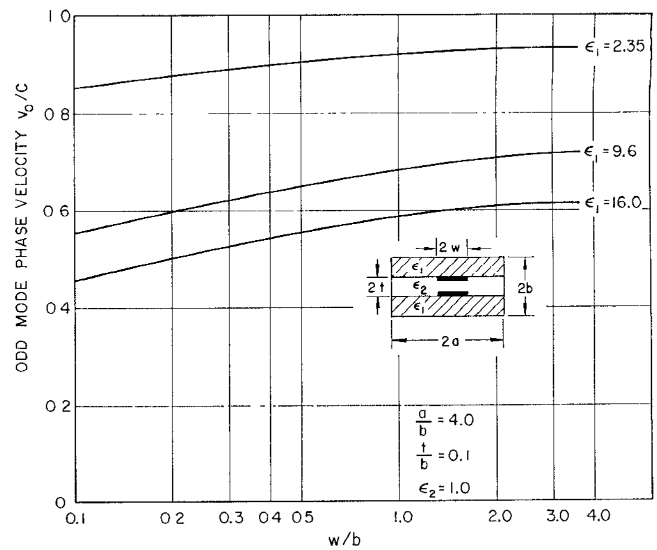

Fig. 16. Normalized odd-mode phase velocity as a function of (w/b) $[(a/b)=4,\ (t/b)=0.1,\ \epsilon_2=1.0]$.

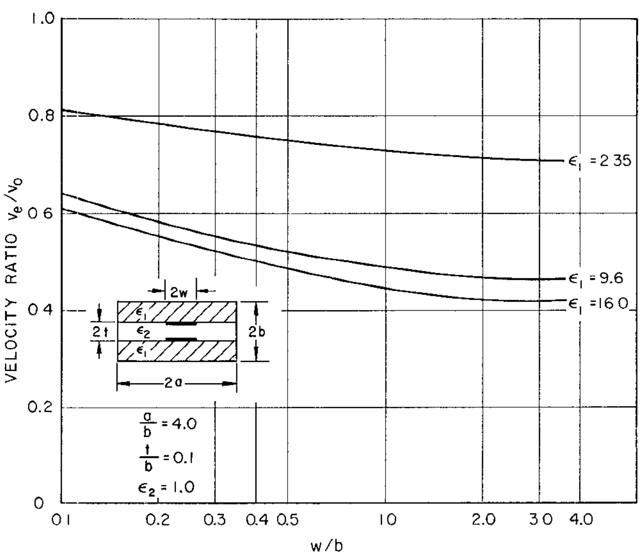

Fig. 15. Ratio of even- to odd-mode phase velocity as a function of (w/b) $[(a/b)=4,\ (t/b)=0.1,\ \epsilon_2=1.0]$.

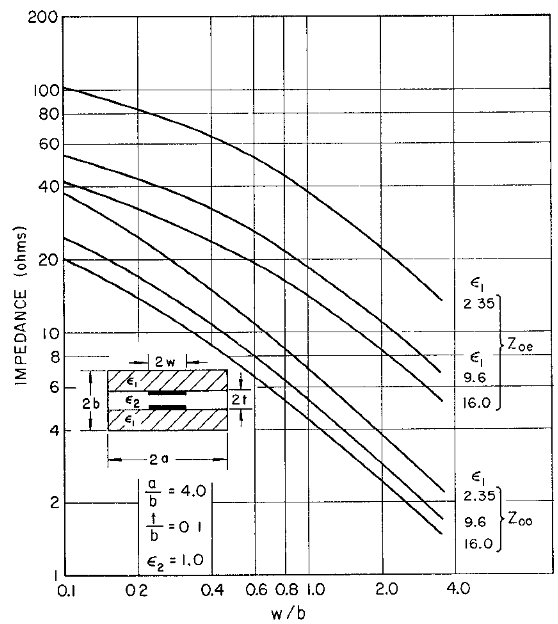

Fig. 17. Odd- and even-mode impedances as a function of (w/b) $[(a/b)=4,\ (t/b)=0.1,\ \epsilon_2=1.0]$.

slab or the width of the center strips increases the phase-velocity ratio and decreases both Z_{0o} and Z_{0e}. Although not shown in these figures, the phase velocities rapidly approach zero as the values of (w/b) and (a/b) become equal (i.e., when the center strips touch the side walls). With wide center strips $[(w/b)=0.8]$ and $\epsilon_2=16$, a velocity ratio of about 3 can be achieved. As shown in Fig. 11, the coupling coefficient increases with both increasing ϵ_2 and center strip widths.

Figs. 12–14 contain velocity and impedance data for the case where the aspect ratio is $(a/b)=1/4$, center-slab thickness $t=0.1\ a$, and air is the upper and lower dielectric layer. In these figures $(w/a)=1.0$ corresponds to the center strips touching the side walls. The velocity ratio has a maximum value of about 2 for $\epsilon_2=16$ and $(w/a)=0.4$.

Figs. 15–18 show velocity, impedance, and coupling coefficient data with aspect ratio $(a/b)=4$, spacing between center strips $t=0.1\ b$, and air as the center dielectric layer. For this case the velocity ratio v_e/v_o is less than 1 and deceases with both increasing ϵ_1 and center-strip widths. With $(w/b)=0.8$ and $\epsilon_1=16$, the velocity ratio is about 0.45. The coupling coefficient decreases with increasing ϵ_1 and increases with increasing center-strip widths.

Figs. 19–22 present velocity, impedance, and coupl-

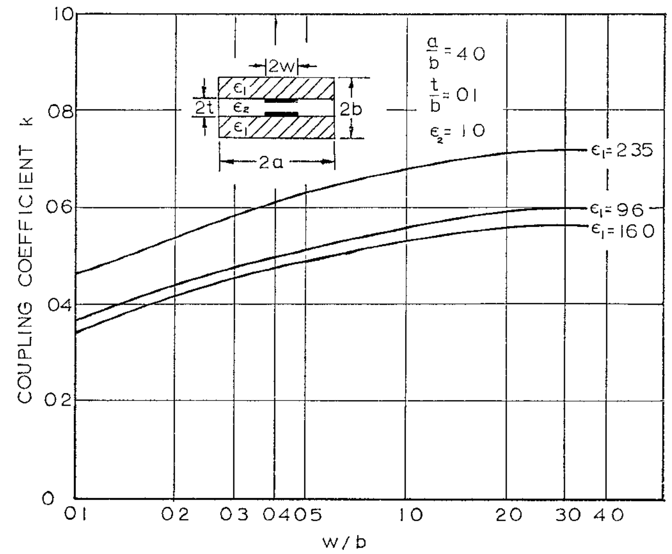

Fig. 18. Coupling coefficient as a function of (w/b) $[(a/b)=4,\ (t/b)=0.1,\ \epsilon_2=1.0]$.

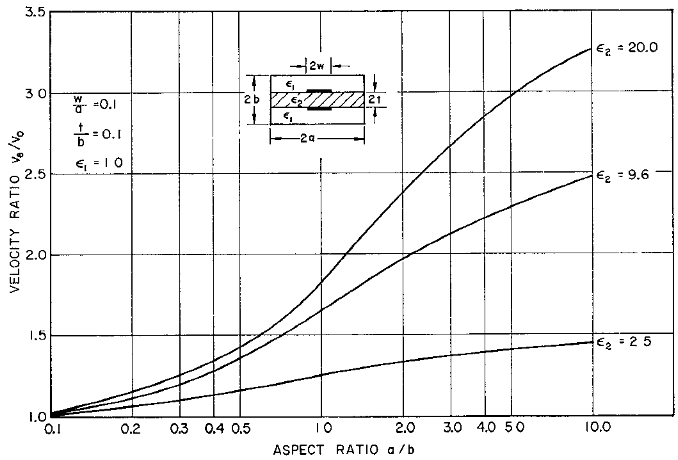

Fig. 19. Ratio of even- to odd-mode phase velocity as a function of aspect ratio (a/b) $[(w/a)=0.1,\ (t/b)=0.1,\ \epsilon_1=1.0]$.

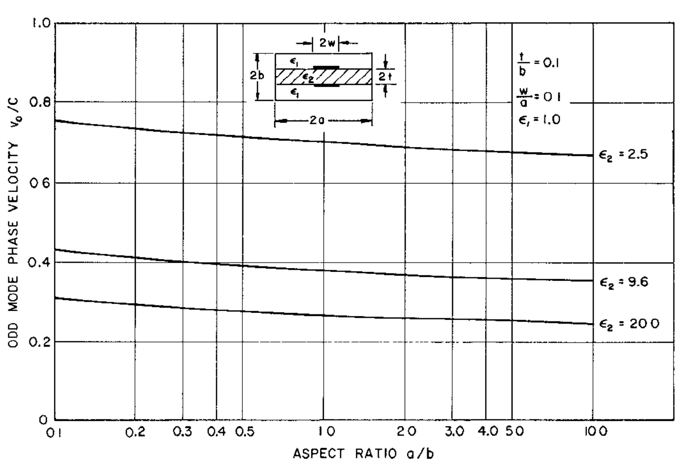

Fig. 20. Normalized odd-mode phase velocity as a function of aspect ratio (a/b) $[(w/a)=0.1,\ (t/b)=0.1,\ \epsilon_1=1.0]$.

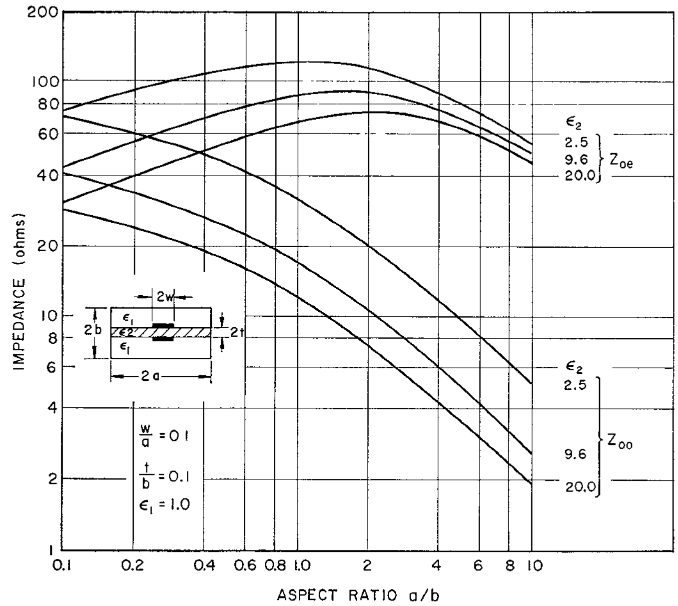

Fig. 21. Odd- and even-mode impedances as a function of aspect ratio (a/b) $[(w/a)=0.1,\ (t/b)=0.1,\ \epsilon_1=1.0]$.

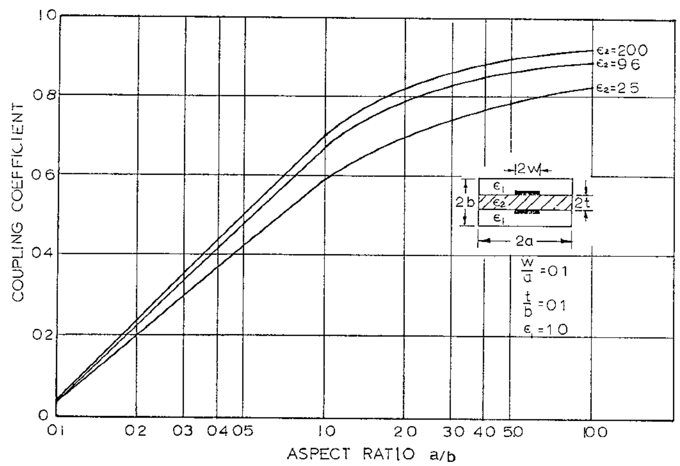

Fig. 22. Coupling coefficient as a function of aspect ratio (a/b) $[(w/a)=0.1,\ (t/b)=0.1,\ \epsilon_1=1.0]$.

ing coefficient data with (w/a) and (t/b) held constant while the aspect ratio (a/b) is varied between 0.1 and 10. With aspect ratio equal to 10 and $\epsilon_2=20$, the velocity ratio is about 3.25. The coupling coefficient varies from almost zero to almost unity as the aspect ratio varies from 0.1 to 10.

Summary and Conclusions

The characteristics of a pair of broadside-coupled strips embedded in a layered dielectric medium and enclosed in a rectangular shield have been presented. With this structure it is possible to achieve large ratios of even- and odd-mode phase velocities. Significant changes occur in the behavior of familiar coupled-line configurations when a large phase-velocity ratio exists, as exemplified by the pronounced bandpass response of the inhomogeneous C section in contrast with the all-pass response of the homogeneous C section.

References

[1] S. B. Cohn, "Shielded coupled-strip transmission line," *IRE Trans. Microwave Theory Tech.*, vol. MTT-3, pp. 29–38, Oct. 1955.

[2] ——, "Characteristic impedances of broadside-coupled strip transmission lines," *IRE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 633–636, Nov. 1960.

[3] H. E. Green, "The numerical solution of some important transmission-line problems," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Microwave Filters*), vol. MTT-13, pp. 676–692, Sept. 1965.

[4] M. K. Krage and G. I. Haddad, "The characteristic impedance and coupling coefficient of coupled rectangular strips in a waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 302–307, May 1968.

[5] T. G. Bryant and J. A. Weiss, "Parameters of microstrip transmission lines and of coupled pairs of microstrip lines," *IEEE Trans. Microwave Theory Tech.* (*1968 Symposium Issue*), vol. MTT-16, pp. 1021–1027, Dec. 1968.

[6] S. V. Judd, I. Whiteley, R. J. Clowes, and D. C. Rickard, "An analytical method for calculating microstrip transmission line parameters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 78–87, Feb. 1970.

[7] J. E. Dalley, "A strip-line directional coupler utilizing a nonhomogeneous dielectric medium," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 706–712, Sept. 1969.

[8] G. I. Zysman and A. Matsumoto, "Properties of microwave C-sections," *IEEE Trans. Circuit Theory*, vol. CT-12, pp. 74–82, Mar. 1965.

[9] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 75–81, Apr. 1956.

[10] D. L. Gish and O. Graham, "Characteristic impedance and phase velocity of a dielectric-supported air strip transmission line with side walls," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 131–148, Mar. 1970.

[11] R. E. Collin, *Field Theory of Guided Waves.* New York: McGraw-Hill, 1960.

[12] R. L. Ketter and S. P. Prauel, Jr., *Modern Methods of Engineering Computation.* New York: McGraw-Hill, 1969.

Computer Analysis of the Fundamental and Higher Order Modes in Single and Coupled Microstrip

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***Abstract*—By means of finite difference methods, dispersion curves are obtained for the fundamental and higher order hybrid modes in both single and coupled microstrip. Structures of realistic proportions are investigated by the use of a graded finite difference mesh. Variational methods are used in deriving the finite difference equations. The higher order modes are found to be similar to LSM slab line modes. A spurious nonphysical class of solutions is found to exist in this and similar formulations, the characteristics of which are described.**

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I. Introduction

IN MODERN microwave devices the integrated circuit is a fundamental component, and microstrip is an essential part of such circuits [1], [2]. Many articles have appeared giving design data for single microstrip [3]–[7], and for pairs of coupled strips [8], but common to all but a few of these publications is the assumption that the fundamental mode of propagation may be approximated by TEM mode propagation (the quasi-static approximation). Because microstrip is a device which contains two different dielectric media, the mode supported can never be TEM (except for dc operation), and in general a hybrid mode propagates. Design based on the quasi-static approximation has often been found to be adequate for the fundamental mode when considering operation below about 4 GHz with low permittivity substrates (κ below 6). However, recent developments require the operation of micro-